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COMMENT ON SLIP VELOCITY AT A FLUID-SOLID BOUNDARY

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The assumption that fluid velocity near a solid boundary is equal to boundary velocity, i.e., the no-slip assumption, is argued to be physically untenable. The assumption is critically examined firstly for dilute gases and we show that the no-slip assumption is incompatible with even the most elementary kinetic theoretical considerations. The effective viscosity coefficient relating shear stress exerted by the solid on the gas to the velocity gradient in the gas differs from the viscosity coefficient applicable to the bulk of the gas. Qualitative arguments are used for liquids and similar conclusions are reached.

A macroscopic model, applicable to both gases and liquids, consistent with the physical requirement of nonzero slip, is then proposed. The shear stress at the boundary is then interpreted as a frictional force proportional to the slip velocity and viscosity is assumed to have the same constant value everywhere.

KEY WORDS: Kinetic theory, shear stress.

I INTRODUCTION

It has long been known, since the time of Maxwell and Helmholtz, that a fluid in contact with a solid may slip past with nonzero velocity at the surface. However, the absence of supporting experimental evidence has led scientists to the conclusion that a slip velocity does not generally exist.^{1,2} Goldstein,³ in his classic treatise on hydrodynamics, has surveyed various propositions on the matter and concluded that slip, if it takes place, is too small or, a quasi-solid layer of fluid, if there is one, is too thin to either be observed or to make an observable difference in the results of theoretical calculations. There exists, however, no compelling theoretical argument supporting the non-slip assumption, despite its wide application in many areas of fluid science and engineering. In fact, we contend that the reverse is true: Physical reasoning points to nonzero slip as a necessary condition for a fluid to exert a shear stress on a solid.³ In the context of the kinetic theory of gases, at least, the necessity of slip seems to have been established, eg., by Kramers and Kistemaker,⁴ although such acknowledgement is by no means universal. For liquids, it is almost considered an article of faith that slip does not occur. We hope that the present article, like the preceding note³ on the same topic, will serve to dispel this misunderstanding.

Schnell⁵ reported some earlier experiments supporting the existence of slip at a fluid-solid boundary, in which water flowing over glass, made water-repellent through

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treatment with the vapour dimethyldichlorosilane silicone, is found to exhibit a small but nevertheless measurable slip velocity. Experiments⁶ on the spreading of liquid films on rotating discs have been satisfactorily interpreted in terms of nonzero slip velocity. Koplík *et al.*⁷ have carried out numerical simulations of fluid flow through a channel and have found that, for a two-fluid system, the no-slip condition appears to break down at the line of contact.

We believe that any macroscopic theory based upon the assumption of zero velocity at the boundary is inconsistent with the above experimental evidence and is in any case conceptually at fault.³ A liquid in contact with a solid surface cannot experience a retarding, tangential force from the solid if there is no relative motion. Unlike the solid-solid case, stationary liquid in contact with a solid surface cannot sustain shear. In making these observations, it is assumed that intermolecular forces are of short range, so that solid molecules therefore effectively interact only with those fluid molecules in the immediate vicinity.

Shear stress in a fluid is a result of a momentum flux, as represented by the tensor 2τ . For a fluid of N particles, each of mass m , located in volume V , the flux of the x -component of momentum in the z -direction is given by^{8,9}

$$\tau_{xz} = nm \overline{v_x v_z} + \frac{nN}{2} \overline{zF_x}, \quad (1)$$

where $n = N/V$ is the number density of molecules, \mathbf{v} is the velocity of a fluid particle and

$$\mathbf{F} = -\nabla\psi \quad (2)$$

is the intermolecular force, where ψ is the potential energy of a given pair of molecules. The overhead bar denotes an appropriate average. We have in mind a system of coordinates in which the z -axis is defined by the normal to the boundary and x -axis by the direction of the average fluid flow velocity, $\bar{\mathbf{v}} = (\bar{v}_x, 0, 0)$. The rhs of (1) vanishes for equilibrium distribution functions. Non-vanishing contributions arise from spatial inhomogeneities, for example, a shear $\partial\bar{v}_x/\partial z$ in fluid velocity, and then appropriate non-equilibrium distribution functions¹⁰ must be employed. The viscosity coefficient, η , of a fluid, defined by

$$\tau_{xz} = -\eta \frac{\partial\bar{v}_x}{\partial z}, \quad (3)$$

can be found in this way, at least in the bulk of the fluid. However, near the boundary, modifications to the above expression for τ_{xz} will arise and the effective viscosity coefficient, η' , will be a property of the boundary as well as of the fluid.

For a rarefied gas, a typical molecule spends most of its time as a free particle between occasional, abrupt collisions. Transport of momentum is therefore accomplished chiefly by each molecule carrying its own momentum. This is represented by the first term on the rhs of (1). The second term makes negligible contributions, because ψ is vanishingly small for interparticle spacing characteristic of gases. Green¹¹ and Mori¹² have shown by means of a collision approximation that the above averaging is consistent with Chapman-Enskog theory.¹³

In marked contrast, the second term on the rhs of (1) is dominant for liquids. There, each molecule is effectively enclosed in a cell formed by its nearest neighbours and only occasionally migrates to a neighbouring cell. A molecule is therefore predominantly under the influence of its nearest neighbours and momentum is transferred through space via forces exerted between neighbouring molecules. In the next two sections we consider both cases using straightforward calculations and arguments.

So long as one is far from the boundary, the first term on the rhs of (1) is relatively straightforward to calculate, eg., by using Chapman-Enskog theory to solve Boltzmann's equation.¹³ Boundary effects, however, lead to very difficult problems in kinetic theory^{14,15} and for this reason, we have in Section II adopted an approach consistent with the semi-quantitative scope of our work, namely, the elementary method of mean free paths. Although crude in some respects, the method does bring out quite clearly the essential features of our argument concerning slip at the boundary. The approach is similar to that found in well known texts by Huang¹⁶ and McDaniel.¹⁷

Liquid properties, on the other hand, are not so easy to model. In Section III we give a formal derivation of the stress in a fluid, starting from (1), devoting particular attention to the boundary region but otherwise following Ref. 18. Results analogous to the case of a rarefied gas are obtained. We discuss the results and further implications in Section IV.

II GASES

a) Mean free path analysis near boundaries

For simplicity, we take a plane boundary surface, the normal to which defines the z -axis of a system of coordinates (Fig. 1). All properties are assumed uniform in the x - y plane.

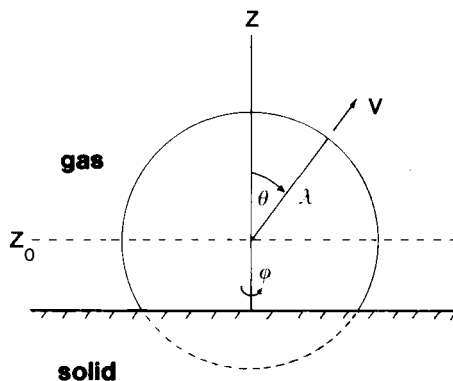


Figure 1 Molecules passing through the plane $z = z_0$ have on the average made a collision at $z = z_0 \mp \lambda \cos \theta$, where λ is the mean free path between collisions, depending upon whether they are moving in the $\pm z$ direction. Close to the boundary ($z_0 < \lambda$), the upward flux will contain a contribution from the boundary for angles $\theta < \theta_0 = z_0/\lambda$.

We calculate the transport of an arbitrary property $u(z)$ of the gas through the plane $z = z_0$, using arguments similar to those found in elementary discussions on gas kinetic theory^{16,17}, but allowing for the influence of the boundary surface at $z = 0$. The crux of the argument is that molecules of mean free path λ crossing the plane $z = z_0$ at angle θ to the normal carry with them properties of the gas at $z = z_0 - \lambda \cos \theta$ if moving in the positive z -direction and properties appropriate to $z = z_0 + \lambda \cos \theta$ if moving in the negative z -direction. Without going into the algebraic details we have then that the flux of property u (per particle of fluid) in the positive z -direction is

$$J_u^{(+)}(z_0) = \frac{n\bar{v}}{2} \int_{\theta_0}^{\pi/2} \cos \theta \sin \theta u(z_0 - \lambda \cos \theta) d\theta + J_u^{(B)}, \tag{4}$$

where n the number density of molecules, \bar{v} their average speed and θ_0 is an angle defined by

$$\theta_0 = \begin{cases} \cos^{-1}(z_0/\lambda), & 0 \leq z_0 < \lambda \\ 0, & \lambda < z_0 < \infty \end{cases} \tag{5}$$

The last term $J_u^{(B)}$ on the rhs of (4) accounts for the flux of u due to particles coming directly from the boundary and requires a knowledge of boundary properties for its complete specification. For the present, all we need to know is that

$$J_u^{(B)} = 0, z_0 > \lambda. \tag{6}$$

The next step in the traditional analysis is to approximate $u(z_0 - \lambda \cos \theta)$ by the first two terms in a Taylor series expansion, and hence (4) becomes approximately

$$J_u^{(+)}(z_0) = \frac{1}{4}n\bar{v} \cos^2 \theta_0 [u(z_0) - \frac{2}{3}\lambda \cos \theta_0 u'(z_0)] + J_u^{(B)}. \tag{7}$$

which beyond the ‘‘boundary layer’’ becomes, by virtue of (5) and (6),

$$J_u^{(+)}(z_0 > \lambda) = \frac{1}{4}n\bar{v} [u(z_0) - \frac{2}{3}\lambda u'(z_0)]. \tag{8}$$

The flux in the negative z -direction can be found similarly and is approximately

$$J_u^{(-)}(z_0) = \frac{1}{4}n\bar{v} [u(z_0) + \frac{2}{3}\lambda u'(z_0)] \tag{9}$$

for all values of z_0 , independently of boundary effects.

The *net* flux of u in the z -direction is thus

$$J_u(z_0) = J_u^{(+)}(z_0) - J_u^{(-)}(z_0) = \begin{cases} -\frac{1}{3}n\lambda\bar{v}u'(z_0), & z_0 > \lambda \\ -\frac{1}{4}\bar{v}n(1 - \cos^2 \theta_0)u(z_0) - \frac{1}{6}n\lambda\bar{v}(1 + \cos^3 \theta_0)u'(z_0) + J_u^{(B)}, & 0 \leq z_0 < \lambda \end{cases} \tag{10a}$$

$$\tag{10b}$$

Equation (10a) is the familiar elementary flux–gradient relationship, applicable in the bulk of the fluid. Equation (10b) also furnishes a flux–gradient relationship, if u can be expressed in terms of u' , formally,

$$u = \beta u' \tag{11}$$

in the boundary layer, where β is some property of the gas plus boundary.

b) *Viscous flow*

The flux of x -momentum in the z direction may be found by setting

$$u = m\bar{v}_x, J_u = \tau_{xz} \tag{12}$$

in (10a,b), where m is molecular mass and \bar{v}_x is the x -component of average fluid velocity. Thus, using (5) and (10), we have

$$\tau_{xz}(z_0) = \begin{cases} -\eta \left. \frac{d\bar{v}_x}{dz} \right|_{z_0}, & z_0 > \lambda \tag{13a} \\ -\frac{3}{4}\eta(1 - z_0^2/\lambda^2) \frac{\bar{v}_x(z_0)}{\lambda} - \frac{1}{2}\eta(1 + z_0^3/\lambda^3) \left. \frac{d\bar{v}_x}{dz} \right|_{z_0}, & 0 \leq z_0 < \lambda \tag{13b} \end{cases}$$

where we have assumed that the boundary is a perfect absorber of momentum and have taken the last term on the rhs of (10b) as zero (see discussion under (iii) below), and

$$\eta \equiv \frac{1}{3}nm\lambda\bar{v} \tag{14}$$

is the shear viscosity coefficient of the gas. Equation (13a) applies to the *bulk* of the gas only. Further assumptions are required to make further progress.

(i) *No slip assumption*

If it is assumed that $\bar{v}_x = 0$ (formally equivalent to setting $\beta = 0$ in (11)) in the boundary layer, then (13b) yields

$$\tau_{xz} = -\eta' \frac{d\bar{v}_x}{dz}, \tag{15}$$

where

$$\eta' = \frac{1}{2}\eta(1 + z_0^3/\lambda^3) \tag{16a}$$

is an effective viscosity coefficient. Clearly, the no-slip condition is inconsistent with the assumption of constant viscosity coefficient throughout the fluid. Moreover, the no-slip assumption appears very much *ad hoc*, even in the context of the present, very elementary theory.

(ii) *General boundary condition (11)*

If (11) is assumed with $u = m\bar{V}_x$, then we once again obtain an equation of the form (15) with

$$\eta' = \frac{1}{2}\eta \left(1 + \frac{3}{2} \frac{\beta}{\lambda} \right) \tag{16b}$$

at the boundary.

(iii) *Perfect absorber assumption*

Let us momentarily return to the general discussion in terms of an arbitrary property u , later to be identified with $m\bar{v}_x$, the mean momentum of a fluid particle. Let us also assume that the boundary is perfect absorber of u , so that in (10b)

$$J_u^{(B)} = 0. \quad (17)$$

This is the "boundary condition" to be imposed upon the fluid flow and is the only extra information which may be used to furnish the required expression (11). Consider, as McDaniel¹⁷ has done (without saying so explicitly), that the actual boundary surface $z = 0$ is replaced by a "filter" which transmits the property u in one direction (downwards) only and that the gas now occupies all of space $-\infty < z < \infty$. In that case, the flux from the boundary $J_u^{(B)}$ is the gas flux for $0 \leq \theta < \theta_0$ and corresponds to the "missing part" of the integral in (4), ie.,

$$J_u^{(B)} = \frac{1}{2} n \bar{v} \int_0^{\theta_0} \cos \theta \sin \theta u(z_0 - \lambda \cos \theta) d\theta \quad (18)$$

and this must be equal to zero by virtue of (17). Proceeding through the usual Taylor expansion we find then, for $z_0 = 0$ ($\theta_0 = \pi/2$),

$$\begin{aligned} J_u^{(B)} &= \frac{1}{4} n \bar{v} (u - \frac{2}{3} \lambda u') \\ &= 0 \end{aligned}$$

from which we find

$$u(0) = \frac{2}{3} \lambda \left. \frac{du}{dz} \right|_0 \quad (19)$$

ie., in terms of equation (11), $\beta = \frac{2}{3} \lambda$. Thus, the flux of u to the boundary is

$$J_u = -\frac{1}{3} n \bar{v} \lambda \frac{du}{dz}, \quad (20)$$

at $z_0 = 0$.

This result can be shown to be true throughout the boundary layer. Thus, with the condition (19) relating the property with its gradient, the flux-gradient relationship is the same (compare (10a) and (20)) throughout the fluid, ie., the same (constant) transport coefficient can be applied everywhere.

If we now specify the property as

$$u = m\bar{v}_x$$

then we arrive at the flux-gradient relationship

$$\tau_{xz} = -\eta \frac{d\bar{v}_x}{dz},$$

where η is given by (14), which now applies throughout the gas, with the boundary condition

$$\bar{v}_x = \frac{2}{3}\lambda \frac{d\bar{v}_x}{dz}(z_0 = 0) \quad (21)$$

relating the (non-zero) slip velocity with the velocity shear.

We would expect similar results to follow from more rigorous kinetic theoretical analysis, although numerical constants would, of course, differ, eg., the coefficient in (21) would be of order λ , although the numerical factor would differ from $2/3$.

Note that McDaniel's analysis¹⁷ is for gaseous charged particles near electrodes and that the property u corresponding to his work is the number density n of these particles. He obtains an expression similar to (21) linking the value of n at the boundary with the density gradient and discusses the result in terms of "extrapolation length", the distance into the boundary to which the density profile extrapolates to zero. A similar concept could have been introduced in connection with fluid flow. We wish at this stage merely to point out that there is a one-to-one correspondence between the existence of a nonzero slip velocity at the boundary of a fluid flow and the nonvanishing of the charged particle density at the surface of an electrode.

There has been some discussion in the literature¹⁹ concerning the "bounce back" boundary condition, in which a molecule reverses its velocity upon impact with the boundary. It has, it seems, been proposed, not because it reflects the reality of boundary collisions in any way (far from it!) but chiefly because it leads to a zero slip velocity. As the no-slip condition itself is of dubious pedigree since it must be imposed *ad hoc*, rather than on physical grounds, there seems little justification for considering this boundary condition further. Likewise, specular (perfectly elastic) reflection at a boundary may well lead to zero slip,¹⁹ but then it obviously also leads to zero tangential momentum-transfer (and hence zero stress) to the boundary! Steady state can never be reached under these circumstances.

III LIQUIDS

In a liquid, the first term on the rhs of (1) can be neglected in comparison with the second, which contains in effect a summation over all pairs of molecules. The actual force experienced by a given molecule is the resultant of forces exerted by all other molecules. Under equilibrium conditions this summation vanishes.

Following well known methods,¹⁸ the average in (1) is calculated by integrating zF_x over a density function $\rho(\mathbf{r}) = \rho(r, \theta, \phi)$, where $\mathbf{r} = (x, y, z)$ is the vector describing the separation of two molecules. We assume that for laminar flow with sufficiently small velocity gradient $\partial\bar{v}_x/\partial z$, the density distribution is given by

$$\rho(r, \theta, \phi) = \rho(r) \left[1 + \frac{\partial\bar{v}_x}{\partial z} U(r) \sin \theta \cos \theta \cos \phi \right], \quad (22)$$

where $\rho(r)$ is the equilibrium density and $U(r)$ is a radial function determined by a second-order differential equation^{20,21} involving the intermolecular potential $\psi(r)$. In this case, the second term in equations (1) and (22) yield for the bulk of the liquid

$$\begin{aligned}\tau_{xz} &= -\frac{n}{2} \int_0^{2\pi} d\phi \int_0^\pi d\theta \int_0^\infty dr \frac{\partial\psi}{\partial r} r^3 \rho(r, \theta, \phi) \sin^2 \theta \cos \theta \cos \phi \\ &= -\left\{ \frac{2\pi}{15} \int_0^\infty dr \frac{\partial\psi}{\partial r} r^3 \rho(r) U(r) \right\} \frac{\partial \bar{v}_x}{\partial z}\end{aligned}\quad (23)$$

ie., equation (3) once more with η given by the expression in curly brackets.

Applying similar arguments on the boundary, however, we find that the range of integration of θ extends only to $\pi/2$, since there is fluid only in the upper hemisphere. The potential energy ψ , as predominantly given by nearest neighbour interactions, will now include terms involving liquid-solid molecule interactions. Likewise, $U(r)$, which depends upon ψ , is now influenced by solid as well as fluid molecules. Formally, then, we expect the momentum flux at the boundary to be

$$\tau_{xz} = -\eta' \frac{\partial \bar{V}_x}{\partial z}, \quad (24)$$

where η' depends now upon both fluid and boundary properties and is given by

$$\eta' = \frac{\pi}{15} \int_0^\infty dr \frac{\partial\psi'}{\partial r} r^3 \phi(r) U'(r), \quad (25)$$

where primes denote properties pertaining to the fluid boundary layer. The calculation of these quantities is beyond the scope of this paper and is in any case not necessary for the qualitative conclusions which we wish to draw. We furthermore expect an equation similar to (24) to apply away from the boundary with η' being a function of z (cf. the discussion for gases in Section II).

IV CONCLUDING REMARKS

In sheared fluid flow the fluid exerts a force on the boundary, through a momentum flux, and vice versa. This force F_B has been calculated in Sections II and III, for gases and liquids respectively, and depends on properties of both fluid and solid boundary. It can generally be written as

$$F_B = -\tau_{xz} = \eta' \left. \frac{\partial \bar{v}_x}{\partial z} \right]_0 \quad (26)$$

at the boundary where viscosity coefficient η differs from the usual bulk viscosity coefficient η . The calculation of η' , as distinct from η , is generally quite difficult and therefore we propose a simple, physical macroscopic model:

- (i) Viscosity coefficient is taken as the same constant value, η , throughout the fluid, both in the bulk and in the boundary layer;
- (ii) At the boundary, the fluid has small slip velocity $v_B = \bar{v}_z(0)$, related to the velocity gradient by (cf. (11))

$$v_B = \bar{v}_x(0) = \beta \left. \frac{\partial \bar{v}_x}{\partial z} \right|_{z=0} \quad (27)$$

where β is a parameter with dimensions of length. Combining (26) and (27) gives

$$F_B = \mu v_B, \quad (28)$$

where

$$\mu = \frac{\eta'}{\beta} \quad (29)$$

is a friction coefficient³, dependent upon properties of the fluid, solid and the nature of the surface.

For gases, the friction coefficient is relatively straightforward to calculate, eg., in equation (21) of Section II we found $\beta = \frac{2}{3} \lambda$ for a boundary acting as a perfect absorber of momentum and then

$$\mu = \frac{1}{2} nm\bar{v} \quad (30)$$

Although the theory leading to this result is rather elementary, we nevertheless expect that more rigorous theories will furnish $\beta \sim \lambda$ and that (30) will provide at least the right order of magnitude estimate for μ .

For liquids, equation (27) is yet to be established. We might expect that $\beta \sim a$, the average interparticle spacing and that therefore

$$\mu \sim \eta/a.$$

At this stage, however, we feel it best to regard μ as an empirical parameter.

We also observe that, if the boundary exerts a force opposing fluid flow, then work must be done at the boundary at a rate

$$\frac{dW}{dt} = \mu v_B^2, \quad (31)$$

per unit area. This is converted to heat. An alternative view of this heat generation could be gained from application of the ideas of Section II to calculate energy transfer from the gas to the boundary. Even if the temperatures of the gas and solid are the same, there is a net energy flux (associated with the nonzero average velocity of the gas) to the boundary which is equal to (31). No such energy dissipation is admitted if slip velocity v_B is arbitrarily chosen to be zero.

We hope that this article, though essentially qualitative in scope, will prompt fluid dynamicists and others to reconsider their views on slip at a fluid-solid interface.

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